THE DETERMINATION OF THERMAL CONDUCTIVITY BY THE METHOD OF COAXIAL CYLINDERS

P. M. Stupak, A. M. Aizen, and N. G. Yampol'skii

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An expression is obtained for the thermal-conductivity coefficients of an investigated substance by the method of coaxial cylinders.

When the method of coaxial cylinders is used to determine the coefficients of the thermal conductivity of an investigated substance, the equation for the calculation can be written in the form

$$\lambda_0 = \frac{Q \ln \dfrac{d_2}{d_1}}{2\pi l \Delta T}. \tag{1}$$

Relation (1) is valid in the presence of isothermal conditions on the surfaces of the measuring cylinders; to attain such conditions it is necessary to eliminate the escape of heat from the ends of the cylinders. This can be done by placing protective heaters on the ends of the cylinders, thereby greatly complicating the construction of the apparatus and the performance of the experiment. It is therefore more advantageous to introduce in formula (1) a theoretically determined correction.

For the measuring cell illustrated in Fig. 1, which has on the axis of the internal cylinder a heat source (electric heater) of radius r_0 with volume density $q_0 = Q/\pi r_0^2 l$, the differential equation for the thermal conductivity is

$$\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} = -\frac{q_0}{\lambda_M} \tag{2}$$

TABLE 1. Thermal Conductivity of Water, Benzene, and Toluene

Material	Temperature T, °C	Thermal conductivity coefficient λ, W/m·deg		
		from (13)	from [2]	from [3, 4]
Water	42,6	0,6343	0,6340	0,6346
	44,2	0,6362	0,6350	0,6398
	59,5	0,6557	0,6550	0,6581
	79,1	0,6740	0,6800	0,6734
Benzene	26,0	0,1438	0,1427	0,1440
	42,2	0,1402	0,1404	0,1402
	59,7	0,1359	0,1369	0,1357
	60,6	0,1365	0,1377	0,1351
Toluene	25,4	0,1329	0,1318	0,1336
	40,6	0,1293	0,1289	0,1300
	59,3	0,1257	0,1254	0,1257
	77,3	0,1212	0,1204	0,1215

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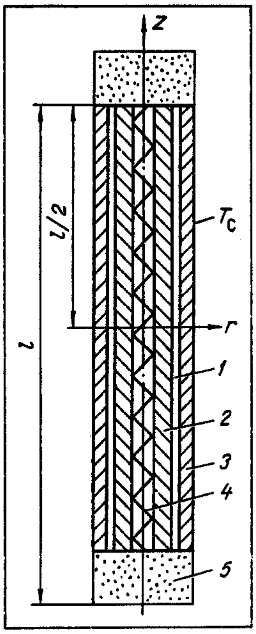

Fig. 1

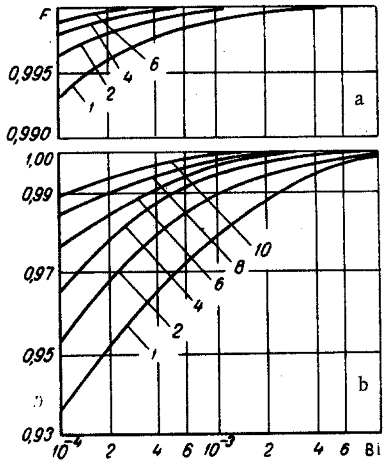

Fig. 2

Fig. 1. Diagram of measuring cell. 1) Investigated substance, 2) internal measuring cylinder, 3) external measuring cylinder, 4) electric heater, 5) multilayer heat insulation.

Fig. 2. Dependence of the correction factor of the Biot number for cells made of copper (a) and Kh18N10T steel (b). The numbers on the curves are the values of l/d_1.

with boundary conditions referred to the internal cylinder

$$(T - T_c)\alpha_1 \pm \lambda_M \left.\frac{\partial T}{\partial z}\right|_{z=\pm\frac{l}{2}} = 0, \qquad (T - T_c)\alpha_2 + \lambda_M \left.\frac{\partial T}{\partial r}\right|_{r=R} = 0. \tag{3}$$

Here $\alpha_1 = 1 \Big/ \sum_{i=1}^{k} \frac{\delta_i}{\lambda_i}$; $\alpha_2 = 1 \Big/ R \left(\frac{1}{\lambda_M} \ln \frac{d_3}{d_2} + \frac{1}{\lambda_0} \ln \frac{d_2}{d_1}\right)$.

The substitution $T - T_c = t$ transforms (2) into an equation in the form

$$\frac{1}{r}\frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial r^2} + \frac{\partial^2 t}{\partial z^2} = -q', \tag{4}$$

$$q' = \begin{cases} \frac{q_0}{\lambda_M} & 0 < r \leqslant r_0 \text{ for heating,} \\ 0 & r_0 < r \leqslant R \text{ for internal cylinder,} \end{cases}$$

and to boundary conditions

$$\alpha_1 t \pm \lambda_M \left.\frac{\partial t}{\partial z}\right|_{z=\pm\frac{l}{2}} = 0, \qquad \alpha_2 t + \lambda_M \left.\frac{\partial t}{\partial r}\right|_{r=R} = 0. \tag{5}$$

To solve the Poisson equation (4) with the boundary conditions (5), we use the method of finite integral transformations [1]. The kernel of the integral transformation needed to eliminate the differential operations with respect to r is determined from the corresponding Sturm–Liouville problem

$$\frac{d}{dr}\left(r\frac{dK}{dr}\right)+\mu rK=0 \tag{6}$$

with boundary conditions

$$K|_{r=0}<\infty,$$
$$K'(R)+\frac{\alpha_2}{\lambda_{\text{M}}}K(R)=0. \tag{7}$$

A solution of (6) exists when $\mu=\mu_n=\gamma_n^2/R^2$, where γ_n are the eigenvalues determined from the equation

$$\text{Bi}\,J_0(\gamma_n)-\gamma_nJ_1(\gamma_n)=0, \tag{8}$$

where

$$\text{Bi}=\frac{\alpha_2R}{\lambda_{\text{M}}}=1\Bigg/\left(\ln\frac{d_3}{d_2}+\frac{\lambda_{\text{M}}}{\lambda_0}\ln\frac{d_2}{d_1}\right)$$

is the Biot number.

Taking into account the solution of Eq. (6), which takes the form

$$K_n=J_0\left(\gamma_n\frac{r}{R}\right),$$

the kernel of the transformation can be written, if condition (8) is satisfied, in the form

$$\Phi_n=r\frac{K_n}{D_n}, \tag{9}$$

where

$$D_n=\int_0^R rJ_0^2\left(\gamma_n\frac{r}{R}\right)dr=\frac{R^2}{2}J_0^2(\gamma_n)\left[1+\left(\frac{\text{Bi}}{\gamma_n}\right)^2\right]. \tag{10}$$

Solving the transform of the equation and then taking the inverse transform, we obtain the following expression for the temperature at any point of the internal cylinder

$$T=\frac{2QR}{\pi lr_0\lambda_{\text{M}}}\sum_{n=1}^{\infty}\frac{J_1\left(\gamma_n\frac{r_0}{R}\right)J_0\left(\gamma_n\frac{r}{R}\right)}{J_0^2(\gamma_n)\left[1+\left(\frac{\text{Bi}}{\gamma_n}\right)^2\right]\gamma_n^3}$$
$$\times\left[1-\frac{\text{ch}\left(\frac{\gamma_n}{R}z\right)}{\text{ch}\left(\gamma_n\frac{l}{d_1}\right)+\frac{\gamma_n}{RH}\text{sh}\left(\gamma_n\frac{l}{d_1}\right)}\right]+T_c, \tag{11}$$

where $H=\alpha_1/\alpha_{\text{M}}$. The specific heat flow at r = R is

$$q=-\lambda_{\text{M}}\frac{\partial T}{\partial r}\Big|_{r=R}=\frac{2Q}{\pi lr_0}\sum_{n=1}^{\infty}\frac{J_1(\gamma_n)J_1\left(\gamma_n\frac{r_0}{R}\right)}{[\gamma_nJ_0(\gamma_n)]^2\left[1+\left(\frac{\text{Bi}}{\gamma_n}\right)^2\right]}$$
$$\times\left[1-\frac{\text{ch}\left(\frac{\gamma_n}{R}z\right)}{\text{ch}\left(\gamma_n\frac{l}{d_1}\right)+\frac{\gamma_n}{RH}\text{sh}\left(\gamma_n\frac{l}{d_1}\right)}\right]. \tag{12}$$

By simple transformations, we obtain the theoretical relation for the thermal-conductivity coefficient of the investigated substance

$$\lambda = \lambda_0 \frac{1}{r_0/R} \sum_{n=1}^{\infty} \frac{4J_1(\gamma_n) J_1\left(\gamma_n \frac{r_0}{R}\right)}{[\gamma_n J_0(\gamma_n)]^2 \left[1 + \left(\frac{\mathrm{Bi}}{\gamma_n}\right)^2\right]} \times \left[1 - \frac{\mathrm{ch}\left(\gamma_n \frac{z_0}{R}\right)}{\mathrm{ch}\left(\gamma_n \frac{l}{d_1}\right) + \frac{\gamma_n}{RH} \mathrm{sh}\left(\gamma_n \frac{l}{d_1}\right)}\right] \tag{13}$$

or

$$\lambda = \lambda_0 F\left(\mathrm{Bi}, \frac{l}{d_1}, \frac{r_0}{R}, \frac{z_0}{R}, RH\right). \tag{13a}$$

Thus, the factor F, which takes into account the heat loss from the ends of the measuring cell, depends on the geometric dimensions of the cell and on the thermal resistance of the insulation on the ends, and also on the Biot number, which in turn depends on the coefficient of thermal conductivity of the material from which the cell is made, and of the investigated substance.

In the absence of heat loss, the factor should be equal to unity, and in the presence of loss it should be smaller than unity, since λ_0 increases in this case as a result of the decrease of ΔT.

Figure 2 shows plots of the correction factor F against the Biot number for different ratios l/d_1, for cells made of copper (λ = 380 W/m·deg) and Kh18N10T steel (λ = 16 W/m·deg).

The remaining parameters were assumed constant for both cases and equal to $r_0/R = 0.4$, R = 5 mm, and $z_0 = 0$; the thermal resistance of the insulation was 0.86 m^2·deg/W.

As seen from the diagram, at equal cell dimensions, the heat loss from the ends depend mainly on the coefficients of thermal conductivity of the cell material and of the investigated substance. Calculations show that the ratio r_0/R exerts practically no influence on the value of the correction factor.

By choosing suitable materials and design for the cell, it is possible to make the heat loss from the ends negligible.

A comparison of the values of the thermal-conductivity coefficients of water, toluene, and benzene, listed in the table and determined from relation (13), and from the data of [2], with the corresponding values of the thermal-conductivity coefficients calculated in the same references, and with the interpolated values for water in accordance with data of [3], and for benzene and toluene in accordance with data of [4], shows that formula (13) yields results having a high degree of accuracy.

NOTATION

Q	amount of heat generated by the electric heater in one hour;
d_1, R	outside diameter and radius of the internal cell;
d_2	inside diameter of external cylinder;
d_3	outside diameter of external cylinder;
l	length of measuring cell;
ΔT	temperature drop in cylindrical layer of investigated substance;
T	temperature;
T_c	temperature of outside surface of outer cylinder;
r, z	cylindrical coordinates;
δ_i	thickness of thermal-insulation layer on the ends of the measuring cell;
λ_i	coefficient of thermal conductivity of the thermal insulation;
k	number of layers of thermal insulation;
λ_M	coefficient of thermal conductivity of the material of the outer and inner cylinders;
J_0, J_1	Bessel functions of zero and first order and of first kind;
z_0	distance from the point where the temperature drop is measured in the layer of the investigated substance to the origin.

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